ASME Journal of Applied Mechanics
Online journal at:
https://asmedigitalcollection.asme.org/appliedmechanics

Magneto-Mechanical Metamaterials: A Perspective

Jay Sim
Department of Mechanical Engineering,
Stanford University,
Stanford, CA 94305
e-mail: jsim10@stanford.edu

Ruike Renee Zhao[1]
Department of Mechanical Engineering,
Stanford University,
Stanford, CA 94305
e-mail: rrzhao@stanford.edu

Magneto-mechanical metamaterials possess unique and tunable properties by adjusting their shape configurations in response to an external magnetic field. Their designs and functionalities are diverse and are utilized in a wide variety of applications, such as highly tunable elastic and electromagnetic wave filters and targeted shape morphing. In this perspective, we examine the general background of magneto-mechanical metamaterials and their diverse applications. The possible future directions in the field are also thoroughly discussed. [DOI: 10.1115/1.4063816]

Keywords: magneto-mechanical metamaterial, magnetic actuation, active metamaterial, mechanical properties of materials, structures, wave propagation

Metamaterials, originating from the Greek word "meta," meaning "beyond," are a type of manmade material that feature unique properties and capabilities. Their distinguishing characteristics lie in their structure, which consists of repeating unit cells. The properties of these materials are primarily governed by the geometry of the unit cell rather than its material composition and are not observed in natural materials. Active metamaterials are a subcategory of metamaterials and possess unit cells capable of modifying their geometry and, correspondingly, their properties, in response to external triggers or stimuli. By manipulating these stimuli, active metamaterials showcase the remarkable ability to demonstrate tunable properties, significantly enhancing their functionality and applicability.

Among the many diverse types of active metamaterials, the magneto-mechanical metamaterial possesses unique advantages for shape reconfiguration and property tuning through the application of an external magnetic field, a rapid, reversible, and untethered actuation method. Figure 1(*a*) demonstrates the general mechanism of magneto-mechanical metamaterials. Oftentimes, sections of the unit cell of a magneto-mechanical metamaterial have specifically designed magnetization directions. When subjected to an external magnetic field, typically generated by permanent magnets or electromagnetic coils, the magnetized sections of the magneto-mechanical metamaterial experience magnetic torque, which leads to shape transformation into an actuated mode. This process is reversible and following the removal of the magnetic field, or in some cases, an application of a reversed magnetic field, the magneto-mechanical metamaterial returns to its initial mode. In addition, there are two strategies to fabricate magneto-mechanical metamaterials. The first option is embedding magnetic particles into a soft polymeric material, creating a magnetic soft composite [2,3], and the second involves inserting permanent rigid magnets into a soft or rigid matrix [4,5]. Of these two strategies, magnetic soft composites are a more robust strategy to easily tune the mechanical properties of magneto-mechanical metamaterials [6]. Accordingly, the remainder of this perspective will focus on such.

Figure 1(*b*) illustrates an example of magneto-mechanical metamaterials using a square-shaped unit cell, whose magnetization distribution leads to a compact configuration under the applied downward external magnetic field [1]. An opposite magnetic field applied to the compact metamaterial can bring it back to the initial mode. Figure 1(*c*) depicts experimental images of the fabricated magneto-mechanical metamaterial and its actuation based on the unit cell design in Fig. 1(*b*). Here, an 8×8 array of unit cells deforms to the compact configuration when a uniform downward magnetic field is applied. By exploiting their fast and programmable shape morphing capability, magneto-mechanical metamaterials have also found an expansive range of functionalities and are popular candidates in a variety of applications, as shown in Fig. 1(*d*). One of the common applications is in wave manipulation. The reconfigurable geometry of a magneto-mechanical metamaterial affects the propagation of electromagnetic [7], elastic [1,8–11], and acoustic/elastic [12,13] waves, allowing them to be utilized as filters to block the transmission of waves at specific frequencies or as waveguides to guide waves in paths along the overall metamaterial. Moreover, the structural reconfiguration of the unit cell can directly tune the mechanical properties, such as structural stiffness and Poisson's ratio, of a magneto-mechanical metamaterial [4,14–18]. The shape-reconfigurable magneto-mechanical metamaterials can also serve as a structural framework for soft robots [19–22]. Figure 1(*e*) presents an experimental image of area-preservation [1] in magneto-mechanical metamaterials. Here, the magneto-mechanical metamaterial achieves unique shape reconfiguration modes while maintaining consistent overall dimensions. This is critical in applications where constant area coverage of substrates is required such as a conformal reconfigurable antenna. Selective actuation, which means only certain unit cells undergo shape change while others are left unchanged, is an effective strategy that facilitates the programmability of magneto-mechanical metamaterials, as shown in Fig. 1(*f*). This increases the number of possible

[1]Corresponding author.

Contributed by the Applied Mechanics Division of ASME for publication in the Journal of Applied Mechanics. Manuscript received October 3, 2023; final manuscript received October 8, 2023; published online October 31, 2023. Assoc. Editor: Pradeep Sharma.

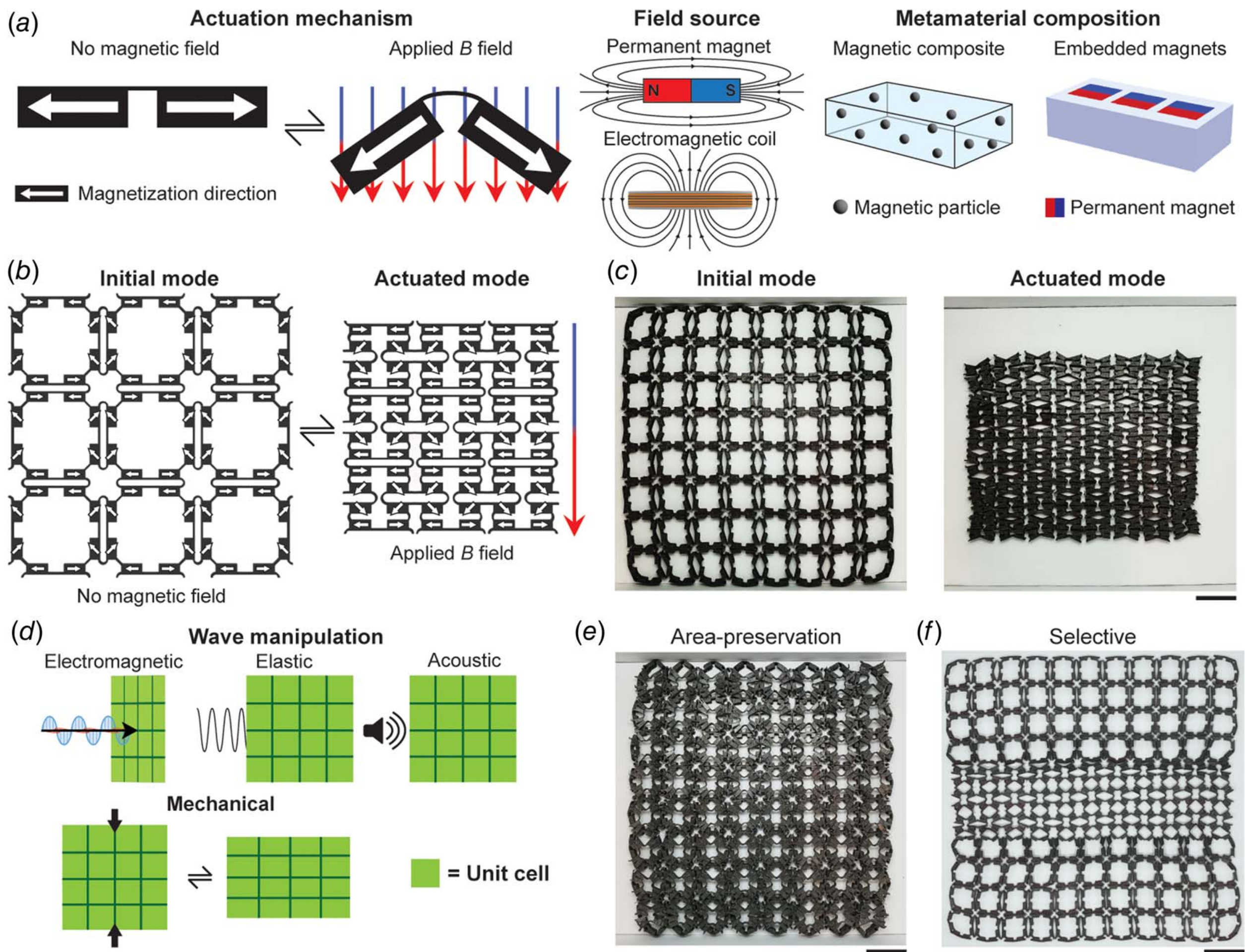

Fig. 1 Mechanism and applications of magneto-mechanical metamaterials. ((*b*), (c), and (e) Reproduced from Sim et al. [1] with permission from Wiley-VCH GmbH). (*a*) Actuation mechanism of a magneto-mechanical metamaterial with magnetic field sources and fabrication methods. (*b*) Schematic of a magneto-mechanical metamaterial and its actuation. (*c*) Experimental actuation of magneto-mechanical metamaterial. Scale bar: 20 mm. (*d*) Applications of magneto-mechanical metamaterials in electromagnetic, elastic, and acoustic wave manipulation, and mechanical property tuning. (e) Functional shape reconfiguration of magneto-mechanical metamaterials with constant overall dimensions and area-preservation. Scale bar: 20 mm. (*f*) Demonstration of selective actuation where the middle three rows are actuated. Scale bar: 30 mm.

shape reconfiguration modes in the metamaterial for diverse functionality and applicability.

To demonstrate the diversity and versatility of magneto-mechanical metamaterials, Fig. 2 includes four examples of functional magneto-mechanical metamaterials for different applications. Figure 2(*a*) depicts a magneto-mechanical metamaterial that uses its shape reconfiguration ability as a tunable electromagnetic wave filter [7]. In its deployed mode where no magnetic field is present, there is a bandgap from 24.5 to 26 GHz where electromagnetic waves at these frequencies are filtered. But when a magnetic field is applied to reconfigure its shape to the folded mode, the metamaterial allows waves at all frequencies to pass. Furthermore, magneto-mechanical metamaterials as tunable filters are not limited to electromagnetic waves. Figure 2(*b*) shows an elastic wave filtering magneto-mechanical metamaterial [1]. Here, the metamaterial consists of two stacked metamaterial arrays with each array having opposite magnetization directions, causing the stacked arrays to experience magnetic attraction to one another and the flipped magnetizations allow for independent actuation of layers. The combination of magnetic attraction and independent actuation is the key principle in achieving area-preservation in this magneto-mechanical metamaterial. In response to an external magnetic field, only one layer will undergo shape reconfiguration and the other layer provides area-preservation, and the attraction between the two layers holds them together into a single metamaterial system. Subsequently, the metamaterial is employed as an elastic waveguide and as a wave filter for tunable frequencies. Mechanical property tuning is another important application of magneto-mechanical metamaterials, as shown in Fig. 2(*c*) [15]. This metamaterial is fabricated by 3D printing two different magnetically responsive materials, with one being a soft material and the other being a magnetic shape memory polymer whose stiffness is tuned via heat. By controlling the external thermal source, the stiffness of specific sections of the metamaterial is tuned to activate and deactivate the magnetic field-induced shape morphing such that the metamaterial will show either a positive or negative Poisson's ratio during deformation. Finally, magneto-mechanical metamaterials are utilized as the structural framework for soft robots, as exampled by the "starfish" metamaterial robot in Fig. 2(*d*) [19]. Under a magnetic field, the five metamaterial "legs" contract toward the body center. Once the field is removed, the legs elongate back to the original length.

In recent years, there have been great advances in the field of magneto-mechanical metamaterials. However, to expand the application spectrum of this active material system for rich programmability and property tunability, there are still numerous avenues for future works on their actuation strategies, advanced manufacturing methods, and design methods [23]. In this section, we outline our views on five potential future directions and unsolved challenges of magneto-mechanical metamaterials.

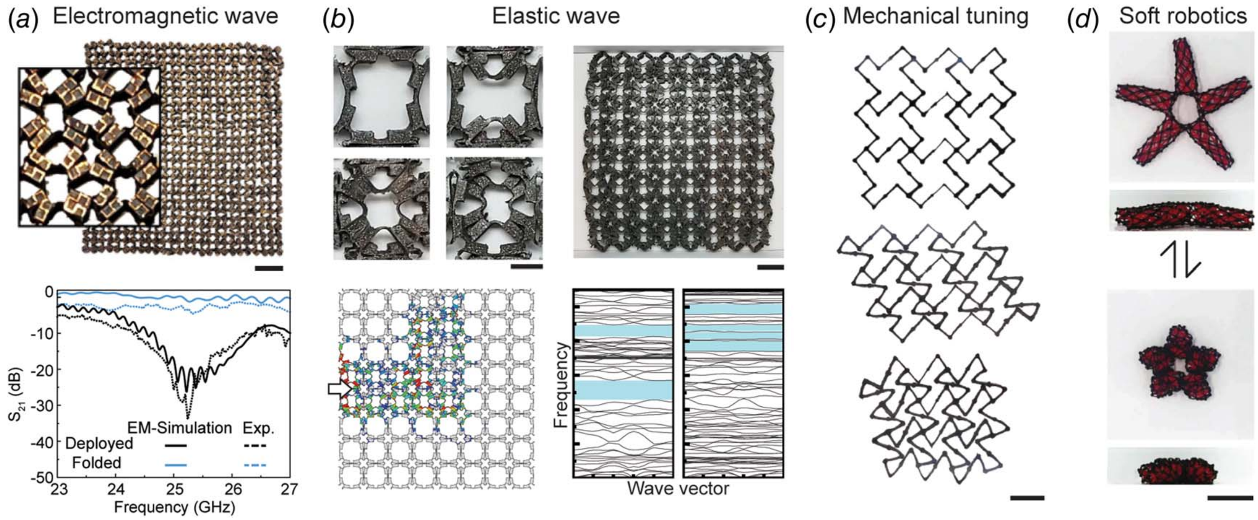

Fig. 2 Four examples of magneto-mechanical metamaterials. (*a*) Magneto-mechanical metamaterial utilized as a conformal electromagnetic wave filter. (Reproduced from Wu et al. [7]. with permission from Wiley-VCH GmbH). Scale bar: 20 mm. (*b*) Area-preserving bilayer magneto-mechanical metamaterial with applications in elastic wave control and waveguide. Scale bars left to right: 5 mm and 20 mm. (Reproduced from Sim et al. [1] with permission from Wiley-VCH GmbH). (c) 3D printed magneto-mechanical metamaterial with the ability to demonstrate either positive or negative Poisson's ratio. (Reproduced from Ma et al. [15] with permission from American Chemical Society). Scale bar: 10 mm. (*d*) Tensegrity-based magneto-mechanical metamaterial with applications in soft robotics. (Reproduced from Lee et al. [19] with permission from the American Association for the Advancement of Science). Scale bar: 50 mm.

1 Integration of Shape Morphing and Locking of Magneto-Mechanical Metamaterials

Most existing works on magneto-mechanical metamaterials feature rapid, reversible, and untethered actuation. They reconfigure their shape when the magnetic field is applied and revert immediately after the field is removed. However, sometimes, it is desirable for magneto-mechanical metamaterials to maintain their shape reconfigurations, even when the magnetic field is removed. Recently, two strategies, structural snapping and material stiffness tuning, have been explored for magneto-mechanical metamaterials to maintain their shape. Structural snapping utilizes the snap-through behavior of slender beams and shells to create multiple stable states while material locking uses a material's property change in response to external stimuli. Both Pal et al. and Chen et al. [24–26] demonstrate the combination of snap-through behavior with magnetic field-responsive materials to create bistable magneto-mechanical metamaterials. After the application of a magnetic field, metamaterials snap-through to new shape configurations and remain locked after the field is removed. Upon the application of a reversed magnetic field, the metamaterial can reverse back to the initial mode. Alternatively, Ze et al. introduce shape-locking magneto-mechanical metamaterials by way of magnetic shape memory polymers (M-SMPs) [27]. M-SMP softens and stiffens when heated and cooled, respectively. Consequently, a magneto-mechanical metamaterial fabricated from M-SMP achieves shape transformation in its soft state. Then, the magnetic field is removed once the metamaterial cools and stiffens, locking the actuated mode in place. To return to the initial mode, the metamaterial requires another heating cycle. Both approaches, structural snapping and material stiffness tuning, provide repeatable and stable shape configurations but beyond these two methods, it would be beneficial to explore other shape-locking mechanisms in magneto-mechanical metamaterials to expand the application.

2 Enhancing Programmability of Shape Reconfigurations

The programmability of magneto-mechanical metamaterials determines their applicability, and the programmability is directly related to the number of shape reconfigurations. Selective actuation is a strategy in which only certain sections of a magneto-mechanical metamaterial are actuated and is a potential option to vastly increase the number of shape reconfigurations. Shape locking, as described earlier, can be combined with selective actuation as a new programming method. Pal et al. physically move permanent magnets toward a magneto-mechanical metamaterial, snapping some unit cells into a new shape reconfiguration [25] and skipping others to leave them in their initial configuration. Chen et al. utilize small electromagnetic coils to generate local magnetic fields such that the magnetic field is essentially applied to only one unit cell at a time [26]. Currently, selective actuation has only been demonstrated in planar magneto-mechanical metamaterials. Strategies to expand selective actuation to the three-dimensional space with more complex metamaterial structures are highly desirable but underexplored. For example, revolving structures have extensive electromagnetic applications [28], and applying selective actuation to revolving magneto-mechanical metamaterial structures vastly increases their tunability and applicability.

3 Conformal Magneto-Mechanical Metamaterials

Conformal magneto-mechanical metamaterials are highly desired for applications that include highly complex 3D geometries, such as electromagnetic antennae. But, most magneto-mechanical metamaterials have been demonstrated on planar surfaces [10,15,26]. While some magneto-mechanical metamaterials feature actuation on curved substrates [1,7], these surfaces possess relatively low curvature and the metamaterials are only draped on top of the substrate. There are tremendous needs for developing magneto-mechanical metamaterials capable of conforming to complex 3D objects while simultaneously featuring multiple shape reconfigurations. One potential strategy is to design metamaterials with multiple distinct unit cells to accommodate the 3D surface at the metamaterial's undeformed state and during its shape reconfigurations. Optimization-based techniques could be a powerful tool in this endeavor [29], where the unit cells and their distribution can be designed to fulfill the conformability requirement during the actuation of the metamaterials. Regardless of approach, magneto-mechanical metamaterials capable of completely conforming to complex 3D geometries have not been achieved and, once demonstrated, have the potential to revolutionize the field.

4 Advanced Manufacturing of Magneto-Mechanical Metamaterials

Further work into the advanced manufacturing of magneto-mechanical metamaterials is another promising direction to expand the tunability. Since the size of the unit cell determines the operating frequency of acoustic and electromagnetic waves, higher frequency waves necessitate a reduction in the size of the unit cell [14,30]. Therefore, it is worth investigating advanced manufacturing methods of submillimeter magneto-mechanical metamaterial unit cells for high-frequency applications. Currently, most magneto-mechanical metamaterials are fabricated by molding and casting [1,7] and are largely limited to the millimeter and centimeter scales. Additive manufacturing is a popular approach to transform the fabrication of magneto-mechanical metamaterials with different types of additive manufacturing being utilized [31]. For example, permanent magnets or electromagnetic coils have been combined with direct ink writing (DIW) [32] and digital light processing (DLP) [15,22,33] printing to program the magnetization alignment of magneto-mechanical metamaterials. In addition, stereolithography (SLA) has also been used to program the magnetization alignment of magnetic field-responsive materials [34], and the same process could be utilized for magneto-mechanical metamaterials. Yet, additive manufacturing is not the only possible direction in the manufacturing of submillimeter magneto-mechanical metamaterials. Electron beam lithography has been utilized to plant nanomagnets onto a silicone matrix [35] for nano-scale magneto-mechanical metamaterials. However, further study on advanced fabrication processes for micro and nano-length scales is worth investigating.

5 Inverse Design and Design Optimization

Most magneto-mechanical metamaterials with tunable properties follow the forward design process, where a metamaterial is first designed and fabricated, often through a trial-and-error approach, and then studied for its properties and functionalities, by experimental methods, finite element analysis (FEA), or theoretical predictions. But, with the emergence of machine learning and optimization-based techniques, specific properties are preselected, and machine learning or optimization is used to generate the corresponding geometries. Strategies such as deep learning algorithms [33], topology optimization [36], and evolutionary algorithms [37] have been recently explored and combined with computational structural analysis such as FEA, beam models, and structural experiments, to generate magneto-mechanical metamaterial geometries with predetermined behaviors or properties such as targeted shape morphing [38], desired global strains, and Poisson's ratio [33]. The inverse designs are usually achieved by pre-programming the shape of the unit cell, the magnetization distributions, the actuating magnetic field, and the location of actuation for the magneto-mechanical metamaterial. Although advances have been made with applying machine learning and optimization-based techniques to inverse design processes of magneto-mechanical metamaterials, the focus has predominantly been limited to guiding their shape change and deformation. However, as these techniques continue to evolve, targeting more ambitious properties becomes a possibility. For instance, electromagnetic, phononic, and acoustic wave manipulation poses a highly complicated and nonlinear problem but offers appealing functionalities with extensive applications. Using machine learning and optimization-based methods to engineer magneto-mechanical metamaterials capable of filtering these waves at specific target frequencies can lead to a new archetype of highly tunable wave manipulation devices. It is also important to note that previous examples have been confined to two-dimensional space. There is significant potential in extending these methods to encompass 3D shape change and deformation.

In summary, this perspective discusses the fundamental mechanisms underlying magneto-mechanical metamaterials and provides recent examples where they have been employed in applications spanning wave manipulation, mechanical property tuning, and soft robotics. Furthermore, we discuss potential avenues for future research endeavors in this field. In the authors' opinion, the field of magneto-mechanical metamaterials is inherently boundless, with the possibilities primarily dictated by the geometry and shape of the unit cell. Future directions depend on the creativity of unit cells and their corresponding design methodologies. Additionally, despite the recent surge in interest in magneto-mechanical metamaterials, it is worth noting that the field is emerging and much remains to be explored in terms of manufacturing strategies, design approaches, and applications.

Acknowledgment

R.R.Z. and J.S. acknowledge the support from NSF Award ECCS-2300157 and NSF Award CMMI-2142789.

Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

No data, models, or code were generated or used for this paper.

References

[1] Sim, J., Wu, S., Dai, J., and Zhao, R. R., 2023, "Magneto-Mechanical Bilayer Metamaterial With Global Area-Preserving Density Tunability for Acoustic Wave Regulation," Adv. Mater., **35**(35), p. 202303541.
[2] Kim, Y., Yuk, H., Zhao, R., Chester, S. A., and Zhao, X., 2018, "Printing Ferromagnetic Domains for Untethered Fast-Transforming Soft Materials," Nature, **558**(7709), pp. 274–279.
[3] Jiralerspong, T., Bae, G., Lee, J. H., and Kim, S. K., 2020, "Wireless Control of Two- and Three-Dimensional Actuations of Kirigami Patterns Composed of Magnetic-Particles-Polymer Composites," ACS Nano, **14**(12), pp. 17589–17596.
[4] Gu, H., Boehler, Q., Ahmed, D., and Nelson, B. J., 2019, "Magnetic Quadrupole Assemblies With Arbitrary Shapes and Magnetizations," Sci. Rob., **4**(35), pp. 1–8.
[5] Zou, B., Liang, Z., Zhong, D., Cui, Z., Xiao, K., Shao, S., and Ju, J., 2023, "Magneto-Thermomechanically Reprogrammable Mechanical Metamaterials," Adv. Mater., **35**(8), p. 2207349.
[6] Qi, J., Chen, Z., Jiang, P., Hu, W., Wang, Y., Zhao, Z., Cao, X., et al., 2022, "Recent Progress in Active Mechanical Metamaterials and Construction Principles," Adv. Sci., **9**(1), p. 2120662.
[7] Wu, S., Eichenberger, J., Dai, J., Chang, Y., Ghalichechian, N., and Zhao, R. R., 2022, "Magnetically Actuated Reconfigurable Metamaterials as Conformal Electromagnetic Filters," Adv. Intell. Syst., **4**(9), p. 2200106.
[8] Dudek, K. K., Iglesias Martínez, J. A., Ulliac, G., Hirsinger, L., Wang, L., Laude, V., and Kadic, M., 2023, "Micro-Scale Mechanical Metamaterial With a Controllable Transition in the Poisson's Ratio and Band Gap Formation," Adv. Mater., **35**(20), p. 2210993.
[9] Zhang, Q., Cherkasov, A. V., Xie, C., Arora, N., and Rudykh, S., 2023, "Nonlinear Elastic Vector Solitons in Hard-Magnetic Soft Mechanical Metamaterials," Int. J. Solids Struct., **280**, p. 112396.
[10] Montgomery, S. M., Wu, S., Kuang, X., Armstrong, C. D., Zemelka, C., Ze, Q., Zhang, R., Zhao, R., and Qi, H. J., 2021, "Magneto-Mechanical Metamaterials With Widely Tunable Mechanical Properties and Acoustic Bandgaps," Adv. Funct. Mater., **31**(3), pp. 1–10.
[11] Pierce, C. D., Willey, C. L., Chen, V. W., Hardin, J. O., Berrigan, J. D., Juhl, A. T., and Matlack, K. H., 2020, "Adaptive Elastic Metastructures From Magneto-Active Elastomers," Smart Mater. Struct., **29**(6), p. 065004.
[12] Xia, P., Lai, Y., and Liu, X., 2022, "Adjustable Magnetic-Control Design of a Metasurface for Sound Insulation," Front. Mech. Eng., **7**, p. 116.
[13] Zhao, J., Li, X., Wang, Y., Wang, W., Zhang, B., and Gai, X., 2017, "Membrane Acoustic Metamaterial Absorbers With Magnetic Negative Stiffness," J. Acoust. Soc. Am., **141**(2), pp. 840–846.
[14] Galea, R., Dudek, K. K., Farrugia, P. S., Zammit Mangion, L., Grima, J. N., and Gatt, R., 2022, "Reconfigurable Magneto-Mechanical Metamaterials Guided by Magnetic Fields," Compos. Struct., **280**, p. 114921.

[15] Ma, C., Wu, S., Ze, Q., Kuang, X., Zhang, R., Qi, H. J., and Zhao, R., 2021, "Magnetic Multimaterial Printing for Multimodal Shape Transformation With Tunable Properties and Shiftable Mechanical Behaviors," ACS Appl. Mater. Interfaces, **13**(11), pp. 12639–12648.
[16] Xia, N., Jin, D., Pan, C., Zhang, J., Yang, Z., Su, L., Zhao, J., Wang, L., and Zhang, L., 2022, "Dynamic Morphological Transformations in Soft Architected Materials via Buckling Instability Encoded Heterogeneous Magnetization," Nat. Commun., **13**(1), p. 7514.
[17] Wu, S., Ze, Q., Zhang, R., Hu, N., Cheng, Y., Yang, F., and Zhao, R., 2019, "Symmetry-Breaking Actuation Mechanism for Soft Robotics and Active Metamaterials," ACS Appl. Mater. Interfaces, **11**(44), pp. 41649–41658.
[18] Zhang, Q., Cherkasov, A. V., Arora, N., Hu, G., and Rudykh, S., 2023, "Magnetic Field-Induced Asymmetric Mechanical Metamaterials," Extrem. Mech. Lett., **59**, p. 101957.
[19] Lee, H., Jang, Y., Choe, J. K., Lee, S., Song, H., Lee, J. P., Lone, N., and Kim, J., 2020, "3D-Printed Programmable Tensegrity for Soft Robotics," Sci. Rob., **5**(45), pp. 1–12.
[20] Wen, L., Pan, F., and Ding, X., 2020, "Tensegrity Metamaterials for Soft Robotics," Sci. Rob., **5**(45), pp. 3–5.
[21] Li, L., Yao, H., and Mi, S., 2023, "Magnetically Driven Modular Mechanical Metamaterials With High Programmability, Reconfigurability, and Multiple Applications," ACS Appl. Mater. Interfaces, **15**(2), pp. 3486–3496.
[22] Roh, S., Okello, L. B., Golbasi, N., Hankwitz, J. P., Liu, J. A. C., Tracy, J. B., and Velev, O. D., 2019, "3D-Printed Silicone Soft Architectures With Programmed Magneto-Capillary Reconfiguration," Adv. Mater. Technol., **4**(4), p. 1800528.
[23] Jiang, S., Liu, X., Liu, J., Ye, D., Duan, Y., Li, K., Yin, Z., and Huang, Y. A., 2022, "Flexible Metamaterial Electronics," Adv. Mater., **34**(52), p. 2200070.
[24] Chen, T., 2023, "Programming Material Logic Using Magnetically Controlled Bistability," Proc. Natl. Acad. Sci. U. S. A., **120**(17), pp. 10–12.
[25] Pal, A., and Sitti, M., 2023, "Programmable Mechanical Devices Through Magnetically Tunable Bistable Elements," Proc. Natl. Acad. Sci. U. S. A., **120**(15).
[26] Chen, T., Pauly, M., and Reis, P. M., 2021, "A Reprogrammable Mechanical Metamaterial With Stable Memory," Nature, **589**(7842), pp. 386–390.
[27] Ze, Q., Kuang, X., Wu, S., Wong, J., Montgomery, S. M., Zhang, R., Kovitz, J. M., Yang, F., Qi, H. J., and Zhao, R., 2020, "Magnetic Shape Memory Polymers With Integrated Multifunctional Shape Manipulation," Adv. Mater., **32**(4), pp. 1–8.
[28] Schurig, D., Mock, J. J., Justice, B. J., Cummer, S. A., Pendry, J. B., Starr, A. F., and Smith, D. R., 2006, "Metamaterial Electromagnetic Cloak at Microwave Frequencies," Science, **314**(5801), pp. 977–980.
[29] Wang, L., Liu, Z., Da, D., Chan, Y. C., Chen, W., and Zhu, P., 2022, "Generalized De-Homogenization via Sawtooth-Function-Based Mapping and Its Demonstration on Data-Driven Frequency Response Optimization," Comput. Methods Appl. Mech. Eng., **395**, p. 114967.
[30] Dudek, K. K., Martínez, J. A. I., Ulliac, G., and Kadic, M., 2022, "Micro-Scale Auxetic Hierarchical Mechanical Metamaterials for Shape Morphing," Adv. Mater., **34**(14).
[31] Montgomery, S. M., Kuang, X., Armstrong, C. D., and Qi, H. J., 2020, "Recent Advances in Additive Manufacturing of Active Mechanical Metamaterials," Curr. Opin. Solid State Mater. Sci., **24**(5).
[32] Tabrizi, M., Ware, T. H., and Shankar, M. R., 2019, "Voxelated Molecular Patterning in Three-Dimensional Freeforms," ACS Appl. Mater. Interfaces, **11**(31), pp. 28236–28245.
[33] Ma, C., Chang, Y., Wu, S., and Zhao, R. R., 2022, "Deep Learning-Accelerated Designs of Tunable Magneto-Mechanical Metamaterials," ACS Appl. Mater. Interfaces, **14**(29), pp. 33982–33902.
[34] Shinoda, H., Azukizawa, S., Maeda, K., and Tsumori, F., 2019, "Bio-Mimic Motion of 3D-Printed Gel Structures Dispersed With Magnetic Particles," J. Electrochem. Soc., **166**(9), pp. B3235–B3239.
[35] Cui, J., Huang, T. Y., Luo, Z., Testa, P., Gu, H., Chen, X. Z., Nelson, B. J., and Heyderman, L. J., 2019, "Nanomagnetic Encoding of Shape-Morphing Micromachines," Nature, **575**(7781), pp. 164–168.
[36] Zhao, Z., and Zhang, X. S., 2023, "Encoding Reprogrammable Properties Into Magneto-Mechanical Materials via Topology Optimization," npj Comput. Mater., **9**(57).
[37] Wu, S., Hamel, C. M., Ze, Q., Yang, F., Qi, H. J., and Zhao, R., 2020, "Evolutionary Algorithm-Guided Voxel-Encoding Printing of Functional Hard-Magnetic Soft Active Materials," Adv. Intell. Syst., **2**(8), p. 2000060.
[38] Wang, L., Chang, Y., Wu, S., Renee, R., and Chen, W., 2023, "Physics-Aware Differentiable Design of Magnetically Actuated Kirigami for Shape Morphing," https://arxiv.org/abs/2308.05054.